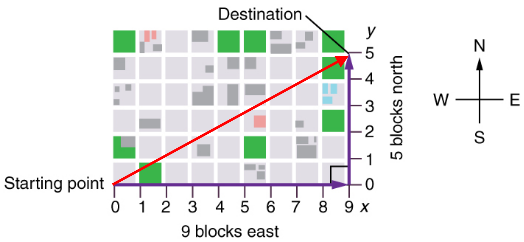
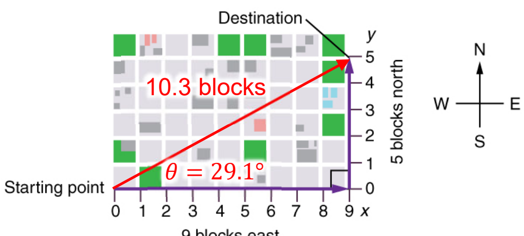
Two Dimensional Motion

- A person walks a two-dimensional path between two points.



- The resulting displacement is the straight-line path between the two points.

- The resulting displacement vector can be calculated using the Pythagorean theorem and trigonometry.

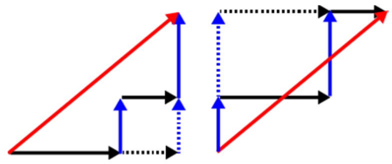


$$\sqrt{9^2 + 5^2} = 10.3$$

$$\theta = \tan^{-1}\left(\frac{5}{9}\right) = 29.1°$$

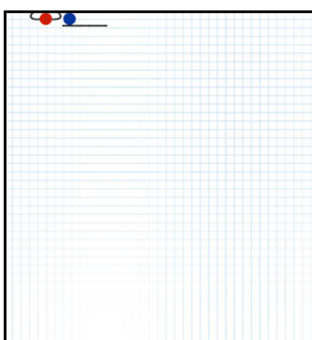
Independence of Motion

- The horizontal and vertical components of two-dimensional motion are independent of each other. Any motion in the horizontal direction does not affect motion in the vertical direction, and vice versa.

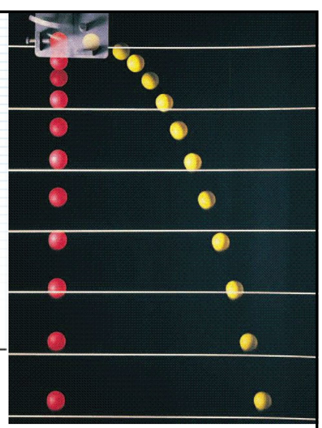


Waifer X (CC BY 2.0)

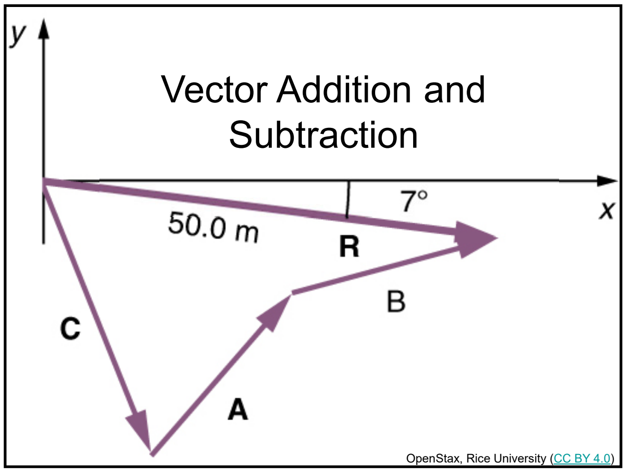
https://youtu.be/zMF4CD7i3hg



OilerLagrangian (CC BY-SA 4.0)

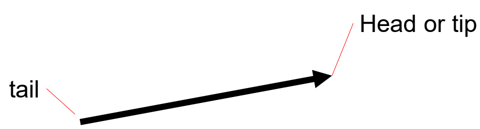


Vector Addition and Subtraction



Graphical Method

- A vector is shown as an arrow of length proportional to the magnitude and pointing in the direction of the vector relative to some reference frame (i.e., coordinate system).



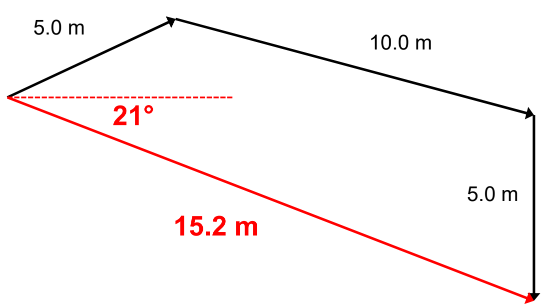
Steps

1. Draw an arrow to represent the first vector using a ruler and protractor.
2. Draw an arrow to represent the second vector. Place the tail of the second vector at the head of the first vector.
3. If there are more than two vectors, continue this process for each vector to be added.

4. Draw an arrow from the tail of the first vector to the head of the last vector. This is the **resultant**, or the sum, of the other vectors.
5. To get the magnitude of the resultant, measure its length with a ruler.
6. To get the direction of the resultant, measure the angle it makes with the reference frame using a protractor.

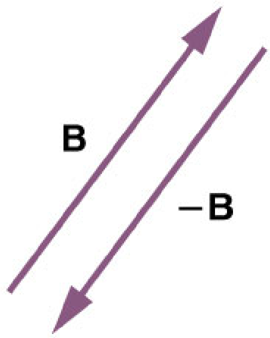
Example

- Suzy walks 5.0 m in a direction 25.0° north of east. Then, she walks 10.0 m heading 15.0° south of east. Finally, she turns and walks 5.0 m directly south. Calculate her displacement.



15.2 m 21° south of east

- To subtract vectors, we reverse the direction of the vector.

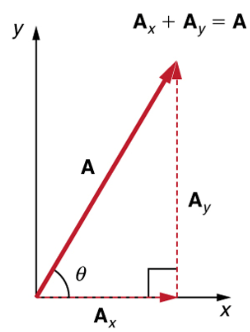


Analytical Method

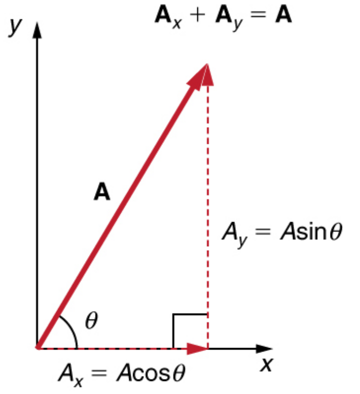
- Vectors will still be represented by arrows of length proportional to the magnitude pointing in the direction of the vector. However, the arrows serve as visual representations only and as such do not have to be to scale.
- Trigonometry and geometry will be used to calculate the resultant vector.

Vector Components

- We can take any vector and separate it into its horizontal (x) and vertical (y) components.

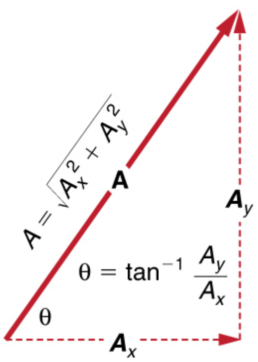


- If the magnitude (its length) and angle θ (its direction) of the vector are known, we can use trigonometry to calculate the components.



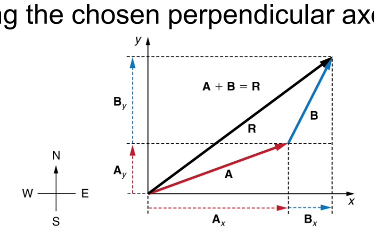
Calculating a Resultant Vector

- If the perpendicular components A_x and A_y of a vector A are known, then the magnitude A and direction θ relative to the x-axis, we can use the Pythagorean theorem and trigonometry.

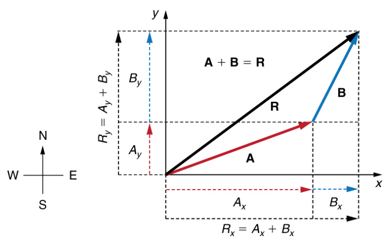


Steps

1. Identify the x- and y-axes that will be used in the problem. Then, find the components of each vector to be added along the chosen perpendicular axes.



2. Find the components of the resultant along each axis by adding the components of the individual vectors along that axis.



3. Calculate the magnitude of the resultant using the Pythagorean theorem.

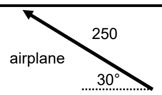
$$R = \sqrt{R_x^2 + R_y^2}$$

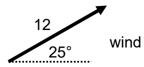
4. Calculate the angle between the x-axis and the hypotenuse using trigonometry.

$$\theta = \tan^{-1}\left(\frac{R_y}{R_x}\right)$$

Example

- An airplane has a velocity of 250 m/s 30° north of west. The wind has a velocity of 12 m/s 25° north of east. Calculate the resulting speed of the airplane (known as ground speed).





<u>x</u>

$-250 \cos 30 = -216.5$

<u>y</u>

$250 \sin 30 = 125$

<u>x</u>

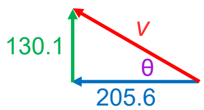
$12 \cos 25 = 10.9$

<u>y</u>

$12 \sin 25 = 5.1$

$x = -216.5 + 10.9 = -205.6$

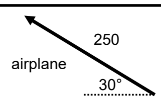
$y = 125 + 5.1 = 130.1$

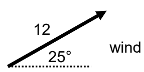


$v = \sqrt{(205.6)^2 + (130.1)^2} = 243.3 \text{ m/s}$

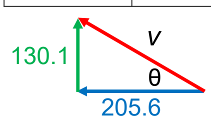
$\theta = \tan^{-1}\left(\frac{130.1}{205.6}\right) = 32.3°$

The resultant velocity is 243.3 m/s 32.3° North of West.





	x-component	y-component
Airplane	$-250 \cos 30 = -216.5$	$250 \sin 30 = 125$
Wind	$12 \cos 25 = 10.9$	$12 \sin 25 = 5.1$
Sum	-205.6	130.1



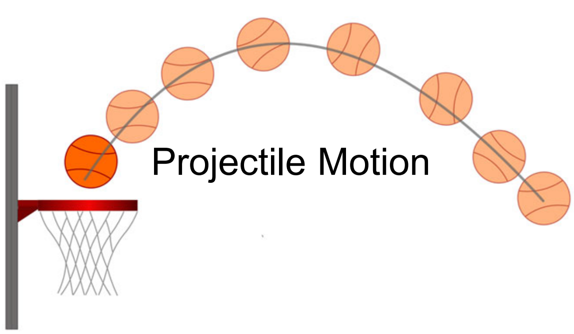
$v = \sqrt{(205.6)^2 + (130.1)^2}$

$v = 243.3 \text{ m/s}$

$\theta = \tan^{-1} \frac{130.1}{205.6}$

$\theta = 32.3°$

The resultant velocity is 243.3 m/s 32.3° North of West.

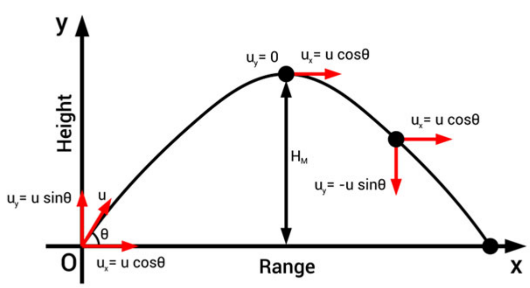


Projectile Motion

LuckySoul (Adobe Stock)

- When an object travels through the air, the vertical motion can be separated from the horizontal motion.
- Gravity affects the vertical motion of the object causing it to accelerate in the vertical direction.

	Horizontal	**Vertical**
Acceleration	No	Yes g, down (-9.8 m/s^2)
Velocity	Constant	Changing



Zizo (Adobe Stock)

Example 1

- A cannon ball is launched with a horizontal velocity of 50 m/s from the top of a 10 m high cliff. Calculate the distance from the bottom of the cliff where the cannon ball lands.

Separate the horizontal and vertical velocities.

- Horizontal
- $v_{x0} = 50\text{ m/s}$
- $a_x = 0$
- $x = ?$
- $t = ?$

- Vertical
- $v_{y0} = 0$
- $a_y = g = -9.8\text{ m/s}^2$
- $y = -10\text{ m}$
- $t = ?$

Solve for time, t, vertically.

- Vertical

$$y = v_{y0}t + \frac{1}{2}a_y t^2$$

$$t = \sqrt{\frac{2y}{g}}$$

$$t = \sqrt{\frac{2(-10\text{ m})}{-9.8\text{ m/s}^2}} = 1.43\text{ s}$$

- The time it takes for the object to fall and hit the ground is the same as the horizontal time.
  - The object stops moving horizontally once the object has hit the ground.
- That means that we can now solve for the horizontal distance.

- Horizontal

$$x = v_{x0}t + \frac{1}{2}a_x t^2$$

$$x = v_{x0}t$$

$$x = (50\ \text{m/s})(1.43\ \text{s}) = 71.5\ \text{m}$$

Example 2

- A cannon ball is launched with a velocity of 50 m/s at an angle of 30° from the horizontal from the top of a 10 m high cliff. Calculate the distance from the bottom of the cliff where the cannon ball lands.

Separate the horizontal and vertical velocities.

- Horizontal
- $v_{x0} = 50\cos30\ \text{m/s}$
- $a_x = 0$
- $x = ?$
- $t = ?$

- Vertical
- $v_{y0} = 50\sin30\ m/s$
- $a_y = -g = -9.8\ \text{m/s}^2$
- $y = -10\ \text{m}$
- $t = ?$

Solve for time, t, vertically.

- vertical

$$y = v_{y0}t + \frac{1}{2}a_y t^2$$

$$-10\ \text{m} = (50 \sin 30\ \text{m/s})t + \frac{1}{2}(-9.8\ \text{m/s}^2)t^2$$

$$4.9t^2 - 25t - 10 = 0$$

$$t = \frac{-(-25) \pm \sqrt{(-25)^2 - 4(4.9)(-10)}}{2(4.9)}$$

$$t = \begin{cases} \cancel{-0.37\ \text{s}} \\ 5.47\ \text{s} \end{cases}$$

- Horizontal

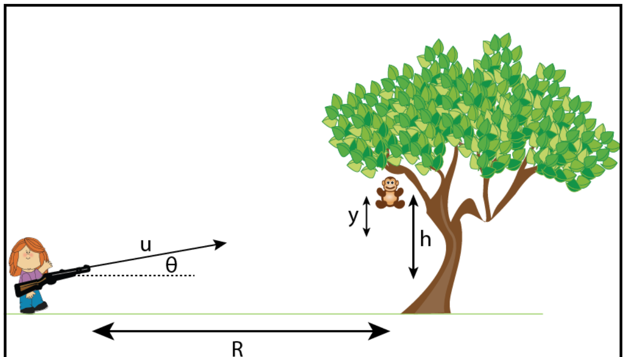
$$x = v_{x0}t + \frac{1}{2}a_x t^2$$

$$x = v_{x0}t$$

$$x = (50 \cos 30\ \text{m/s})(5.47\ \text{s}) = 237\ \text{m}$$

The Monkey and the Hunter

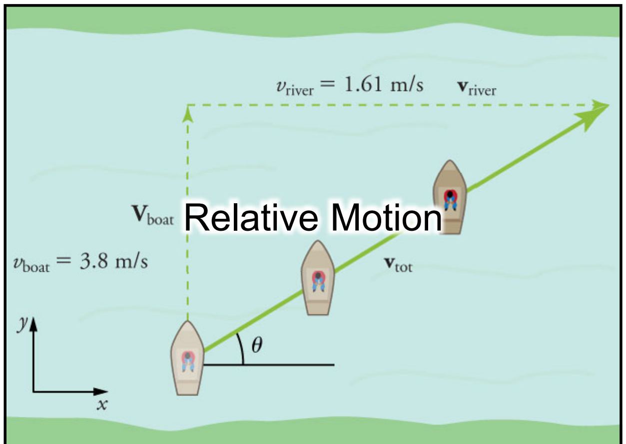
A hunter with a gun goes out in the woods to hunt for monkeys and sees one hanging in a tree. The monkey releases its grip the instant it hears the gun. Where should the hunter aim to hit the monkey?



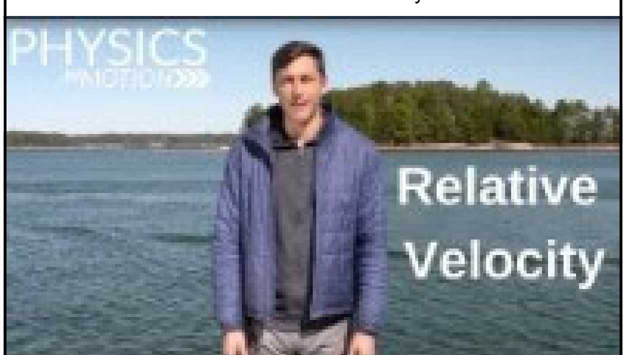
Girl: © MyCuteGraphics.com. Used with permission.
Gun: Firkin – public domain (openclipart.com)
Tree: public domain (openclipart.com)
Monkey: public domain (openclipart.com)

https://youtu.be/CTSwbyCAGHU?t=368

https://youtu.be/cxvsHNRXLjw



What Is Relative Velocity?



https://youtu.be/xBS01yzd0yI

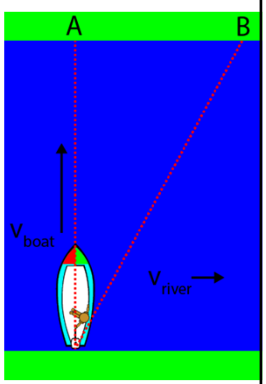
- If a person rows a boat across a rapidly flowing river and tries to head directly for the other shore, the boat instead moves diagonally relative to the shore, because the river carries the boat downstream.
- The boat has a velocity relative to a river and the river has a velocity relative to an observer on solid ground.
- The velocity of the boat relative to the observer is the sum of these velocity vectors.

Example

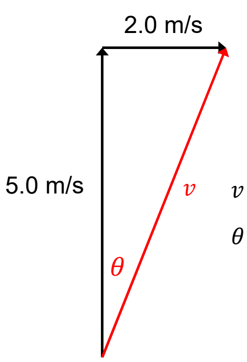
A boat with a velocity of 5.0 m/s is crossing a 50.0 m wide river with a current of 2.0 m/s towards the east.

a) What is the velocity of the boat relative to the shore?

b) What is the distance from A to B?



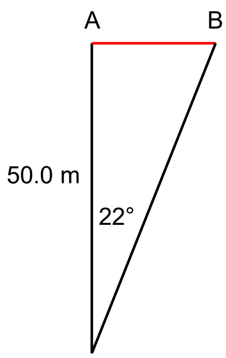
a) Velocity relative to the shore



$$v = \sqrt{5^2 + 2^2} = 5.4\ \mathrm{m/s}$$

$$\theta = \tan^{-1}\left(\frac{2}{5}\right) = 22°\ \text{east of north}$$

b) Distance from A to B



$d = 50 \tan 22 = 20.2 \text{ m}$